Algebra 2

Section 12-2

Matrix Multiplication

Goal: to multiply matrices using scalar and matrix multiplication.

<u>scalar multiplication</u>: multiplying a every element in a matrix by a real number

$$A = \begin{bmatrix} 2 & 5 \\ -3 & 6 \\ -1 & 0 \end{bmatrix} \qquad -3A = \begin{bmatrix} -6 & -15 \\ 9 & -18 \\ 3 & 0 \end{bmatrix}$$

Find -3A

$$A = \begin{bmatrix} 2 & 8 & -3 \\ -1 & 5 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 0 & 5 \\ 0 & 3 & -2 \end{bmatrix}$$

Find 3A - 2B

$$\begin{bmatrix} 6 & 24 & -9 \\ -3 & 15 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 0 & 10 \\ 0 & 6 & -4 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 24 & -19 \\ -3 & 9 & 10 \end{bmatrix}$$

Solve:

$$3X - 2\begin{bmatrix} -1 & 5 \\ 7 & 0 \end{bmatrix} = \begin{bmatrix} 17 & -13 \\ -7 & 0 \end{bmatrix}$$

$$3X - \cancel{\begin{bmatrix} -2 & 10 \\ 14 & 0 \end{bmatrix}} = \begin{bmatrix} 17 & -13 \\ -7 & 0 \end{bmatrix} + \begin{bmatrix} -2 & 10 \\ 14 & 0 \end{bmatrix}$$

$$\cancel{\tfrac{1}{3}}\cancel{3}X = \tfrac{1}{3}\begin{bmatrix} 15 & -3 \\ 7 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 5 & -1 \\ 7/3 & 0 \end{bmatrix}$$

Key Concept Matrix Multiplication

To find element c_{ij} of the product matrix AB, multiply each element in the ith row of A by the corresponding element in the jth column of B. Then add the products.

$$AB = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} -3 & 1 \\ 0 & 2 \end{bmatrix}$

Find AB

$$\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} -3 & 1 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} \underset{r_1c_1}{2\cdot -3 + -1\cdot 0} & \underset{r_1c_2}{2\cdot 1 + -1\cdot 2} \\ \underset{r_2c_1}{3\cdot -3 + 4\cdot 0} & \underset{r_2c_2}{3\cdot 1 + 4\cdot 2} \end{bmatrix}$$

$$\begin{bmatrix} -6 & 0 \\ -9 & 11 \end{bmatrix}$$

Find BA

$$\begin{bmatrix} -3 & 1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} \underset{r_1c_1}{-3\cdot 2 + 1\cdot 3} & \underset{r_1c_2}{-3\cdot -1 + 1\cdot 4} \\ \underset{r_2c_1}{0\cdot 2 + 2\cdot 3} & \underset{r_2c_2}{0\cdot -1 + 2\cdot 4} \end{bmatrix}$$

$$\begin{bmatrix} -3 & 7 \\ 6 & 8 \end{bmatrix}$$

To multiply two matrices, you have to check and see if the dimensions allow for it.

If A is a m x n and B is a n x p matrix, then the product matrix AB is an m x p matrix.

2 x 3   3 x 4

2 x 4

If A is a 4 x 5, and B is 3 x 4 can you multiply AB and BA?

AB

$4 \times \boxed{5 \times 3} \times 4$

No

BA

$3 \times \boxed{4 \quad 4} \times 5$

Yes

3 x 5

Find AB and ~~BA~~, if possible.

2 x 3 ~~x 2~~ x 2

not possible

$A = \begin{bmatrix} -5 & 0 \\ 3 & -2 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 1 & -1 \\ 4 & 2 & 5 \end{bmatrix}$

2 x 2   2 x 3

AB

$$\begin{bmatrix} \underset{r_1c_1}{-5\cdot -2 + 0\cdot 4} & \underset{r_1c_2}{-5\cdot 1 + 0\cdot 2} & \underset{r_1c_3}{-5\cdot -1 + 0\cdot 5} \\ \underset{r_2c_1}{3\cdot -2 + -2\cdot 4} & \underset{r_2c_2}{3\cdot 1 + -2\cdot 2} & \underset{r_2c_3}{3\cdot -1 + -2\cdot 5} \end{bmatrix}$$

$$\begin{bmatrix} 10 & -5 & 5 \\ -14 & -1 & -13 \end{bmatrix}$$

#27.

$$\begin{bmatrix} -1 & 3 & -3 \\ 2 & -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} \underset{r_1c_1}{-5 + 12 + -9} \\ \underset{r_2c_1}{10 + -8 + 3} \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$$

2 x 3   3 x 1

In 1994, suppose a high school player scored 36 two-point field goals and 28 free throws. In 2006, suppose a high school player scored 7 three-point field goals, 21 two-point field goals, and 18 free throws. Using matrix multiplication, how many points did each player score?

$$\begin{array}{cc} & \begin{array}{ccc} 3pt & 2pt & 1pt \end{array} \\ \begin{array}{c} 1994 \\ 2006 \end{array} & \begin{bmatrix} 0 & 36 & 28 \\ 7 & 21 & 18 \end{bmatrix} \end{array} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$2 \times 3 \quad 3 \times 1$

$$\begin{bmatrix} 0 + 72 + 28 \\ 21 + 42 + 18 \end{bmatrix} = \begin{bmatrix} 100 \\ 81 \end{bmatrix}$$

Hwk: pg. 777 - 778

#8, 12, 16 - 26 evens,

29 - 34 all, 38, 40, 46