Algebra II

Section 12-3

Determinants and Inverses

Goal: to find the inverse of a matrix

The inverse of a matrix is used to solve problems involving matrices.

The product of a matrix and its multiplicative inverse will give the multiplicative identity matrix. Not all matrices have inverses.

$4 \cdot \frac{1}{4} = 1$

A square matrix is one where it has the same number of rows as columns.

For a n x n matrix, the multiplicative identity matrix is an n x n matrix I with 1's along the main diagonal and 0's elsewhere.

$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

If A and B are inverses, AB = BA = I

$A = \begin{vmatrix} 1 & 1 \\ 5 & 4 \end{vmatrix}$ $B = \begin{vmatrix} -4 & 1 \\ 5 & -1 \end{vmatrix}$

Are A and B inverses? Yes

AB = I and BA = I

$\begin{bmatrix} 1 & 1 \\ 5 & 4 \end{bmatrix}\begin{bmatrix} -4 & 1 \\ 5 & -1 \end{bmatrix}$

$\begin{bmatrix} -4+5 & 1+-1 \\ -20+20 & 5+-4 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} -4 & 1 \\ 5 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 5 & 4 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Every square matrix has a determinant.

For A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

detA = ad - bc

Ex: $\begin{bmatrix} 3 & 6 \\ -1 & 4 \end{bmatrix}$

detA 12 - -6

detA = 18

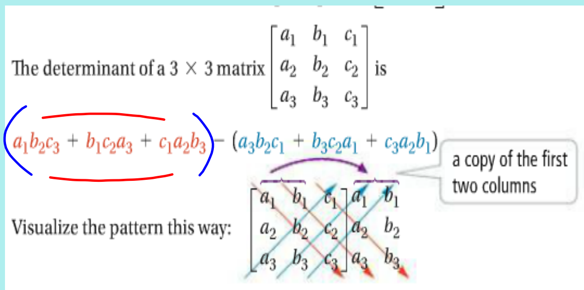
Find the determinant of

A= $\begin{vmatrix} 3 & 6 \\ 2 & 5 \end{vmatrix}$ B= $\begin{vmatrix} -2 & 0 \\ 3 & 0 \end{vmatrix}$

15-12=

detA=3

detB = 0


The determinant of a 3 × 3 matrix $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ is

$(a_1b_2c_3 + b_1c_2a_3 + c_1a_2b_3) - (a_3b_2c_1 + b_3c_2a_1 + c_3a_2b_1)$

Visualize the pattern this way:

Ex: $\begin{vmatrix} 1 & 0 & 3 \\ 2 & 4 & 6 \\ 5 & -1 & 3 \end{vmatrix}$

(12 + -6 + 0) - (60 - 6 0)

6 - 54 = -48

The determinant can help determine in a matrix has an inverse, and if it does exist, to help you find the inverse.

Let A = $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$

If the det A = 0, then the matrix has no inverse. If the detA is not 0 then A^{-1} exists.

$A^{-1} = \frac{1}{\text{detA}} \begin{vmatrix} d & -b \\ -c & a \end{vmatrix}$

Does the following have an inverse? If so, find it.

$A = \begin{vmatrix} 4 & 2 \\ 3 & 2 \end{vmatrix}$ $B = \begin{vmatrix} 2 & 5 \\ -4 & -10 \end{vmatrix}$

$\det A = 8 - 6 = 2$

Yes

$A^{-1} = \frac{1}{2}\begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 1 & -1 \\ -3/2 & 2 \end{bmatrix}$

$\det B = -20 + (+20)$

$\det B = 0$

No Inverse

Find the inverse:

$C = \begin{vmatrix} 7 & 4 \\ 5 & 3 \end{vmatrix}$

$\det C = 1$

Yes

$C^{-1} = \frac{1}{1}\begin{bmatrix} 3 & -4 \\ -5 & 7 \end{bmatrix}$

$C^{-1} = \begin{bmatrix} 3 & -4 \\ -5 & 7 \end{bmatrix}$

Hwk: pg 788 - 789

#8 - 20 (4th), 28 - 32 evens,

42, 44, 52