

Algebra 1

S12-6

Permutations and Combinations

Goal: to find permutations and combinations

On vacation you want to do the following:

- 1 See a ball game
 2 Go to the zoo
 3 Go to the water park
 4 Visit Grandma
- $\left\{ \begin{array}{ll} 1234 & 2134 \\ & 1243 & 2143 \\ & 1432 & 2413 \\ & 1324 & 2431 \\ & 1342 & 2341 \\ & 1423 & 2314 \end{array} \right.$

In how many different orders can you do all of these?

...

You can use counting methods to find the number of possible ways to choose objects without regard to order.

This can be done with an organized list or a tree diagram.

When one event does not affect the result of the second event, they are independent. For independent events, you can use the Fundamental Counting Principle.

If there are m ways to make a first selection and n ways to make a second selection, then there are $m \cdot n$ ways to make the two selections.

How many combinations can you make with 5 shirts and 4 pairs of shorts?

$$5 \cdot 4 = 20$$

For pizza, you can choose thin crust or regular crust, 1 meat topping (from pepperoni, sausage or beef) and 1 vegetable topping (onions, mushrooms, peppers or onions). How many different pizzas could you make?

$$2 \cdot 3 \cdot 4 = 24$$

Other ways to talk about choosing options:

-permutation arrangement in a certain order

Example: electing president, VP and rep

- combination - a selection without regard to order

Finding permutations:

How many ways can you arrange 9 players?

$$9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 362,880$$

From a team with 9 players, how many ways can you pick pitcher, catcher and first base?

$$9 \cdot 8 \cdot 7 = 504 \text{ ways}$$

Other method:

Use the formula

$${}_n P_r = \frac{n!}{(n-r)!}$$

Note: factorial (denoted !) means to multiply that number times every number lower than it.

$${}_{12} P_4 = \frac{12!}{(12-4)!} = \frac{12!}{8!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8!}{8!} = 12 \cdot 11 \cdot 10 \cdot 9 = 11,880$$

Finding combinations, when order doesn't matter is a little more complicated.

- you have to divide out all of the overlap

Example: from 8 sides, you will choose 3

$$\frac{8 \cdot 7}{1 \cdot 2} = \frac{8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3}$$

Formula:

$${}_n C_r = \frac{n!}{r!(n-r)!} = \frac{10!}{6!(10-6)!}$$

Try it:

$${}_{10} C_6 = \frac{10!}{6!(4!)}$$

$$\frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{1 \cdot 2 \cdot 3 \cdot 4 \cdot \cancel{5} \cdot \cancel{6}} = 210$$

Hwk: pg. 766-767

#14 - 18 evens, 26,

28 - 30 evens, 38, 44, 48