Section 3-6

Solving systems with Matrices

Goal: to use Matrices to solve a system of equations

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Systems of equations can be solved using an array of numbers called a *matrix*.
   -write the system without the variables.

A *matrix* is a rectangular array of numbers inside of brackets. The dimensions of the matrix are the number of rows by the number of columns.

$$\begin{bmatrix} 2 & 4 & 3 & 1 \\ 0 & 2 & 5 & -1 \end{bmatrix}$$ This is a 2 x 4 matrix

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$$A = \begin{bmatrix} 4 & -3 & 2 \\ -1 & 2 & 0 \\ 0 & 6 & 5 \end{bmatrix}$$

Each number in the matrix is called a matrix element. These are denoted by their position. 6 is in the position a_{32} because it is in the third row and the second column.

What is matrix element a_{13}?
                            a_{21}?

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Representing a system.
 $x + 2y = -3$
 $3x - y = 5$

The coefficients become the matrix elements.

$$\begin{array}{cc} x & y \\ \begin{bmatrix} 1 & 2 & | & -3 \\ 3 & -1 & | & 5 \end{bmatrix} \end{array}$$

A vertical line replaces the =

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Represent as a matrix:

$3x + y - z = 10$
$0x + 2y + 2z = -5$
$3x + y = -3x + 4$

$$\begin{bmatrix} 3 & 1 & -1 & | & 10 \\ 0 & 2 & 2 & | & -5 \\ 3 & 1 & 0 & | & 4 \end{bmatrix}$$

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What system of linear equations is represented?

$$\begin{array}{cc} x & y \\ \begin{bmatrix} 2 & 5 & | & 3 \\ 0 & -3 & | & -4 \end{bmatrix} \end{array}$$

$2x + 5y = 3$
$-3y = -4$

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Hwk: pg. 179 - 181

#8 - 11 all, 12 - 22 evens,

48 - 51 all, 56 - 58 all

Section 3-6 continued

Solving Systems using

Matrix Operations

Goal: to solve for unknowns in a
system using matrix operations

Solving systems:
  -uses row operations
  -goal is to get to identity matrix

$$\begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{bmatrix}$$

Sol: $x = a$ Sol: $x = a$
        $y = b$ $y = b$
                                     $z = c$

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Row operations:
  -Switch any two rows
  - multiply any row by a constant
  - add one row to another

$$\begin{bmatrix} -3 & 1 & 2 \\ 1 & 1 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & ? \\ 0 & 1 & ? \end{bmatrix}$$

$r_1 \leftrightarrow r_2 \begin{bmatrix} 1 & 1 & 6 \\ -3 & 1 & 2 \end{bmatrix}$ $\frac{1}{4}r_2 \to r_2 \begin{bmatrix} 1 & 1 & 6 \\ 0 & 1 & 5 \end{bmatrix}$

$3r_1 + r_2 \to r_2 \begin{bmatrix} 1 & 1 & 6 \\ 0 & 4 & 20 \end{bmatrix}$ $-r_2 + r_1 \to r_1 \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 5 \end{bmatrix}$
$X = 1$
$Y = 5$

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Solve:

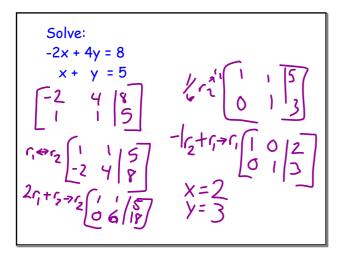$$\begin{bmatrix} 2 & -2 & 20 \\ -6 & 3 & -30 \end{bmatrix}$$

$\frac{1}{2}r_1 \to r_1 \begin{bmatrix} 1 & -1 & 10 \\ -6 & 3 & -30 \end{bmatrix}$

$6r_1 + r_2 \to r_2 \begin{bmatrix} 1 & -1 & 10 \\ 0 & -3 & 30 \end{bmatrix}$

$-\frac{1}{3}r_2 \to r_2 \begin{bmatrix} 1 & -1 & 10 \\ 0 & 1 & -10 \end{bmatrix}$

$r_1 + r_2 \to r_1 \begin{bmatrix} 1x & 0 & 0 \\ 0 & 1y & -10 \end{bmatrix}$

$X = 0$
$Y = -10$

#25. Solve:
  $p - 3q = -1$
  $-5p + 16q = 5$

$$\begin{bmatrix} 1 & -3 & -1 \\ -5 & 16 & 5 \end{bmatrix}$$

$5r_1 + r_2 \to r_2 \begin{bmatrix} 1 & -3 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

$3r_2 + r_1 \to r_1$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$p = -1$
$q = 0$

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Solve:

$-2x + 4y = 8$

$x + y = 5$

$$\begin{bmatrix} -2 & 4 & | & 8 \\ 1 & 1 & | & 5 \end{bmatrix}$$

$\frac{1}{6}r_2 \rightarrow$ $\begin{bmatrix} 1 & 1 & | & 5 \\ 0 & 1 & | & 3 \end{bmatrix}$

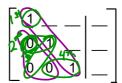$r_1 \leftrightarrow r_2$ $\begin{bmatrix} 1 & 1 & | & 5 \\ -2 & 4 & | & 8 \end{bmatrix}$

$-1r_2 + r_1 \rightarrow r_1$ $\begin{bmatrix} 1 & 0 & | & 2 \\ 0 & 1 & | & 3 \end{bmatrix}$

$2r_1 + r_2 \rightarrow r_2$ $\begin{bmatrix} 1 & 1 & | & 5 \\ 0 & 6 & | & 18 \end{bmatrix}$

$x = 2$

$y = 3$

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In a 3x3 matrix, we will try to get into row echelon form.

$$\begin{bmatrix} 1 & - & - & | & - \\ 0 & 1 & - & | & - \\ 0 & 0 & 1 & | & - \end{bmatrix}$$

#32 from your homework

$\begin{cases} x + y + z = 2 \\ \quad\; 2y - 2z = 2 \\ x \quad\quad - 3z = 1 \end{cases}$

Oct 23-11:43 AM

Hwk: pg 179 - 180
24-32 evens, 33,
38 (use any method),
39, 42, 44
Quiz 3.4-3.6 tomorrow

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