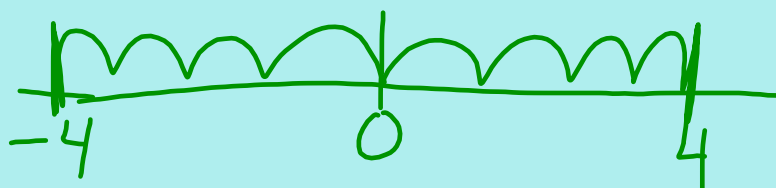# Section 3-7
Absolute Value Equations and Inequalities

| 4 | = 4
   and
|-4 | = 4

This means that when solving absolute value equations and inequalities with unknowns, you have to consider two cases.

Solving absolute value problems start by isolating the absolute value, if needed. Then write a pair of equivalent equations or inequalities.

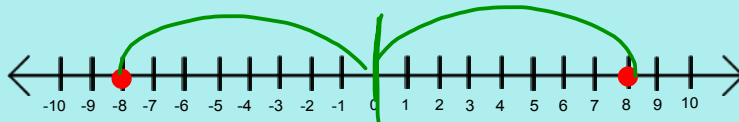What are the solutions to:

$$|x| - 5 = -3$$
$$+5 \; +5$$
$$|x| = 2$$
$$x = 2 \qquad or \quad x = -2$$

$$|3x + 4| = 8$$
$$-8$$

This means that the distance from 3x + 4 to 0 on a number line is 8 units. There are two numbers that are 8 units from 0 on a number line.

Therefore, we have to find the solutions to
$$3x + 4 = 8 \qquad and \qquad 3x + 4 = -8$$
$$-4 \quad -4 \qquad\qquad\qquad -4 \; -4$$
$$\frac{3x = 4}{3} \quad \frac{}{3} \qquad\qquad \frac{3x = -12}{3} \quad \frac{}{3}$$
$$x = \frac{4}{3} \qquad\qquad\qquad x = -4$$

Solve:

$|x + 6| - 3 = 5$

$+3 \quad +3$

$$|x + 6| = 8$$

Case 1 (positive)   **Case 2 (negative)**

$x + 6 = 8$ $x + 6 = -8$

$-6 \quad -6$ $-6 \quad -6$

$x = 2$ ✓ $x = -14$ ✓

Solve:

$2|2x + 6| - 5 = -7$

$+5 \quad +5$

$$\frac{2|2x + 6|}{2} = \frac{-2}{2}$$

$$\boxed{|2x + 6| = -1}$$

$\cancel{\frac{-7}{2}}, \cancel{\frac{7}{2}}$

$\emptyset$

Case 1   **Case 2**

$2x + 6 = -1$ $2x + 6 = 1$

$-6 \quad -6$ $-6 \quad -6$

$\frac{2x}{2} = \frac{-7}{2}$ $\frac{2x}{2} = \frac{-5}{2}$

$x = -\frac{7}{2}$

We will use the same method to solve absolute value inequalities.

Solve:
$|2x + 4| \leq 10$

Case 1   and   Case 2

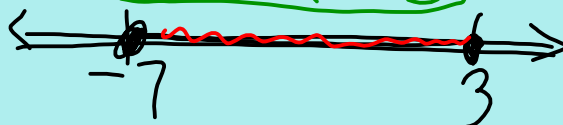$2x + 4 \leq 10$ $2x + 4 \geq -10$

$-4 \quad -4$ $-4 \quad -4$

$\dfrac{2x}{2} \leq \dfrac{6}{2}$ $\dfrac{2x}{2} \geq \dfrac{-14}{2}$

$x \leq 3$ $x \geq -7$

$$-7 \leq x \leq 3$$

Solve:

$|3n - 5| > 4$

$3n - 5 > 4$
$+5 \quad +5$
$\dfrac{3n}{3} > \dfrac{9}{3}$
$n > 3$

$3n - 5 < -4$
$+5 \quad +5$
$\dfrac{3n}{3} < \dfrac{1}{3}$
$n < \dfrac{1}{3}$

A food manufacturer makes 32-oz boxes of pasta. Not every box weighs exactly that, but the allowable difference is at most 0.05 oz. Write and solve an absolute value inequality to find the range of allowable weights.

Hwk: pg. 211 - 212
#12 - 28 (4th), 29, 32 - 42 (4th),
48, 52 - 60 (4th), 64, 66, 76