Section 5-6
The Fundamental Theorem of Algebra

Students will be able to use the Fundamental Theorem of Algebra to solve polynomial equations with complex solutions.

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You can factor any polynomial of degree n into linear factors, but sometimes the factors will involve imaginary numbers.

-The degree of the polynomial tells you how many roots and equation has.

$y = x^2$ has how many roots?

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Find the roots of the following:

$y = x^2 - 9$

$x = \pm 3$

$y = x^2 - 2x + 1$

$0 = (x-1)(x-1)$ $x = 1$ mult. 2

$y = x^2 + 3x + 1$

$x = \frac{-3 \pm \sqrt{9-4}}{2} = \frac{-3 \pm \sqrt{5}}{2}$

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German mathematician Carl Friedrich Gauss is credited with proving the Fundamental Theorem of Algebra:

If P(x) is a polynomial of degree $n \geq 1$, then P(x) = 0 has exactly n roots, including multiple and imaginary roots.

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What are all of the roots of $x^4 + 2x^3 = 13x^2 - 10x$

$0 = x^4 + 2x^3 - 13x^2 + 10x$

$x = -5, 0, 1, 2$

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What are the zeros of
$f(x) = x^4 + 2x^3 - 4x^2 - 7x - 2$?

$x = 2, -1$
$(x-2)(x+1)$

$2 \,|\, 1 \quad 2 \quad -4 \quad -7 \quad -2$
$\quad 2 \quad 8 \quad 8 \quad 2$
$(x-2)$ $1 \quad 4 \quad 4 \quad 1 \quad 0$
$(x^3 + 4x^2 + 4x + 1)$

$(x-2)$ $-1 \,|\, 1 \quad 4 \quad 4 \quad 1$ $(x+1)$
$\quad -1 \quad -3 \quad -1$
$1 \quad 3 \quad 1 \quad 0$
$(x^2 + 3x + 1) \leftarrow x = \frac{-3 \pm \sqrt{5}}{2}$

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Concept Summary:

Equivalent ways to state the Fundamental Theorem of Algebra.

1. Every polynomial of degree n $\geq$ 1 has exactly n roots, including multiple and imaginary roots.
2. Every polynomial of degree n $\geq$ 1 has n linear factors.
3. Every polynomial function of degree n $\geq$ 1 has at least one complex zero.

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Hwk:
pg. 322-323
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Find the Zeros:

$x^5 - x^4 - 7x^3 + 7x^2 - 18x + 18 = 0$

$x = -3, 1, 3$ $(x+3)(x-1)(x-3)(x+i\sqrt{2})(x-i\sqrt{2})$

3	1	-1	-7	7	-18	18
		3	6	-3	12	-18
	1	2	-1	4	-6	0

$x^4 + 2x^3 - x^2 + 4x - 6$

1	1	2	-1	4	-6
		1	3	2	6
	1	3	2	6	0

$(x^3 + 3x^2) + (2x + 6)$

$x^2(x+3) + 2(x+3)$

$(x^2 + 2)(x+3)$ in graph

$x = \pm i\sqrt{2}$

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