Section 8-3
Rational Functions and Their Graphs

Students will be able to:
-identify properties of rational functions
-graph rational functions

You use a ratio of polynomial functions to form a *rational function*, like:

$$y = \frac{x + 3}{x - 2}$$

Rational functions have many different forms of graphs with different features, depending on the functions of the numerator and denominator. Look at the three functions shown on pg. 515.

-notice the hole in graph 2
-notice the dashed blue lines in graph 3
(called asymptotes)
-This makes the two functions **discontinuous**

-The first is a **continuous** function because as you draw it in, your pencil never leaves the paper.

A rational expression is the quotient of two polynomials.

f(x) = is undefined when x = 0. In general, the domain of a rational function is the set of all real numbers except those that make the denominator equal to zero.

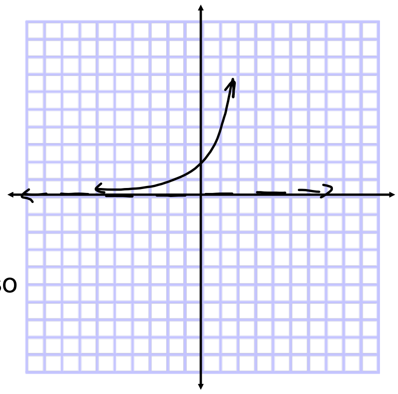
Find the domain of :

$$h(x) = \frac{x^2 - 4x - 21}{x^2 - 9x - 36}$$

Graph $y = 2^x$

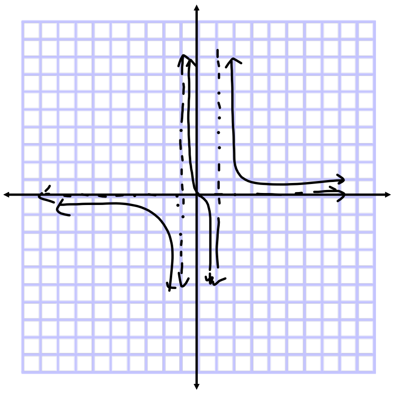
This graph contains a horizontal asymptote.

Some graphs also have vertical asymptotes.



Graph $y = \frac{2x}{x^2 - 1}$

This graph has vertical and horizontal asymptotes.



To find the vertical asymptotes algebraically, use the following information.

If x - a is a factor of the denominator but not a factor of the numerator, then x = a is a vertical asymptote of the graph of the function.

Identify vertical asymptotes:

From previous example: $\frac{2x}{x^2 - 1} = \frac{2x}{(x+1)(x-1)}$

vert asy: $x=-1, x=1$

Horizontal Asymptote:

Let $R(x) = \frac{P}{Q}$, where both P and Q are polynomials.

-If the degree of P and Q are equal and a and b are the leading coefficients, then $y = \frac{a}{b}$ is the equation of the horizontal asymptote.

-If the degree of P < Q, then $y = 0$ is the equation.

-If the degree of P > Q, then there are no horizontal asymptotes.

Identify all vertical and horizontal asymptotes of the following:

$R(x) = \frac{x^3}{x^2 + x - 20} = \frac{x^3}{(x+5)(x-4)}$

vertical: $x = -5,\ x = 4$

horizontal: none

Find the domain of each rational function. Identify all asymptotes.

$d(x) = \frac{3x - 1}{9x^2 - 36} = \frac{(3x-1)}{9(x^2-4)} = \frac{(3x-1)}{9(x+2)(x-2)}$

D: $\mathbb{R}$s, $x \neq \pm 2$

Vert Asym: $x = -2,\ x = 2$

Horiz Asym: $y = 0$

Holes in Graphs:

If x - b is a factor of the numerator and the denominator, then there is a hole in the graph when x = b, unless x is a vertical asymptote.

Identify all asymptotes and holes for:

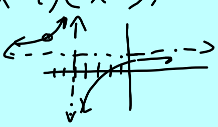
$y = \frac{1x^2 + 3x - 28}{1x^2 + 12x + 35} = \frac{(x+7)(x-4)}{(x+7)(x+5)}$

D: $\mathbb{R}$s $x \neq -7, -5$

Vert: $x = -5$

Horiz: $y = 1$

Holes: $x = -7$



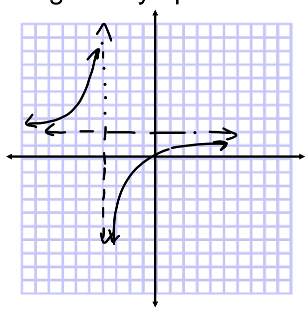
Using Asymptotes to Graph:

Graph $y = \frac{2x - 1}{x + 4}$, showing all asymptotes.

Vert: $x = -4$

Horiz: $y = 2$

Holes: $\varnothing$



Find the domain, asymptotes, holes in the graph. Then graph.

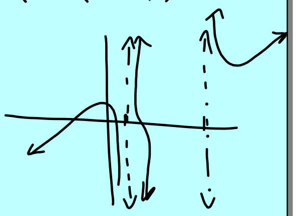
$$m(x) = \frac{\overset{x^3}{x}(x^2 - 4)}{x^2 - 7x + 6} = \frac{x(x+2)(x-2)}{(x-6)(x-1)}$$

D: Rs $x \neq 6, 1$

Vert: $x = 6, x = 1$

Horiz: none

Holes: none



Hwk: pg 521 - 522
18, 21, 24, 29, 41,
43 -44 (must find and show all asymptotes and holes)