Section 7-1
Exploring Exponential Models

Students will be able to model exponential growth and decay.

We have already studied linear functions, where the values of the range change by a fixed amount.

Now we are going to look at exponential functions in which values of the range change by a fixed rate.

$y=2^x$ is an example of an exponential function

Exponential growth is a process that occurs all around us in real life. If you put money in a bank account, it grows exponentially. Cancer cells grow exponentially. The population of the world grows exponentially. Anything that grows at a fixed percent is growing exponentially.

More Examples:

What kinds of things grow exponentially?

- Bacteria population
- Energy resource use
- Number of shopping malls
- Number of automobiles on the freeway
- College tuition (up to a limit)
- Number of Xerox Machines
- Number of cows McDonalds uses each year
- Number of hospital patients
- Number of prisoners
- Number of Web Pages

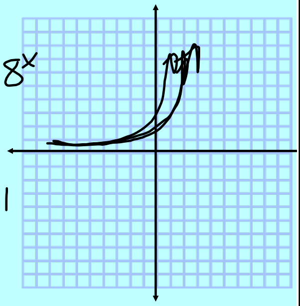
Exponential Function:

$f(x) = ab^x$ where the base, b, is a positive real number other than 1 and x is any real number.

$a \rightarrow$ initial amount
$b \rightarrow$ growth rate factor

-represents repeated multiplication

Graph $f(x) = 2^x$, $g(x) = 5^x$, and $h(x) = 8^x$ on the same graph.

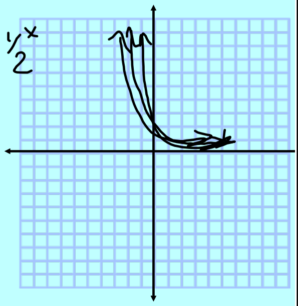


Which on has the fastest growth? the slowest growth? → $y=8^x$

What is the y-intercept? 1

What are the domain and range?
D: Rs
R: $y>0$

Graph $f(x) = (1/2)^x$, $g(x) = (1/5)^x$, and $h(x) =(1/8)^x$ on the same graph.



Which on has the fastest ~~growth~~ decay? the slowest growth? → $y=\frac{1}{2}^x$

What is the y-intercept? 1

What are the domain and range?
D: Rs
R: $y > 0$

Based on your findings, what could we say is a general rule relating the base of an exponential function and what it represents in terms of growth and decay?

Exponential Growth- $b > 1$
Exponential Decay- $0 < b < 1$

Tell whether each function represents exponential growth or decay. Find the y-int.

1). $y = 4(.4)^x$ decay y-int = 4

2). $y = .27(4)^x$ growth y-int = .27

3). $y = 200(2)^{-x} = \frac{1}{2^x} = (\frac{1}{2})^x$ decay, y-int = 200

4). You put $2000 into a college savings account for 4 years. The bank pays 5% interest.
growth y-int = 2000

A bacterial population after n hours can be represented by (with an initial population of 100 bacteria):

(n times)

$100 \times 2 \times 2 \times 2 \ldots \times 2 = 100 \times 2^n$

This is called an exponential expression because the exponent, n, is a variable and the base, 2, is a fixed number. This base is referred to as a growth factor.

To find the growth factor (base) for the rate of growth, you add the percent of increase + 100%.

The population of the US was 248,718,301 in 1990. It was projected to grow at a rate of 9.8% per decade.

$100 + 9.8 = 109.8$

$b = 1.098$

The rate of decay is a decreasing rate, so to find the decay factor you subtract the rate from 100%.

Caffeine leaves an adult's bloodstream at a rate of 15% per hour. Find the decay factor. $100 - 15 = 85\%$ $b = .85$

Find the growth or decay factor for each rate

1. 8.2% decay $100 - 8.2 = 91.8\%$
   $b = .918$

2. 0.075% growth $100 + .075 =$
   $100.075\% =$
   $b = 1.00075$

Predict the population of bacteria for each situation and time period.

39. 75 E. coli bacteria that double every 30 minutes.

a) after 2 hours ← 4 → 30 min periods b) after 3 hours

$y = ab^x$ $y = 75(2)^x$ $y = 75(2)^6$

$y = 75(2)^4 = 1200$ 4800

41. 775 bacteria that triple every hour.

a) after 2 hours b) after 4 hours

$y = 775(3)^x$ $y = 775(3)^4$

$775(3)^2 = 6975$ $= 62{,}775$

Suppose you invest $1500 in a savings account that pays 3.5% annual interest. How much will be in the account after 5 years?

Find b 1st $100 + 3.5 = 103.5$

$b = 1.035$

$y = 1500(1.035)^5 = \$1781.53$

How long will it take it to reach $3000?

$\frac{3000}{1500} = \frac{1500}{1500}(1.035)^x$

$2 = 1.035^x$

Plug + check

$1.41 = 1.035^{10}$

$1.61 = 1.035^{15}$

~~$1.98 = 1.035^{20}$~~

$2.05\ 1.035^{21}$

21 yrs

Hwk: pg. 439 - 440
10, 14, 18 - 25 all,
26 - 30 evens,
34 - 40 evens, 45

Sec6.2NB.notebook